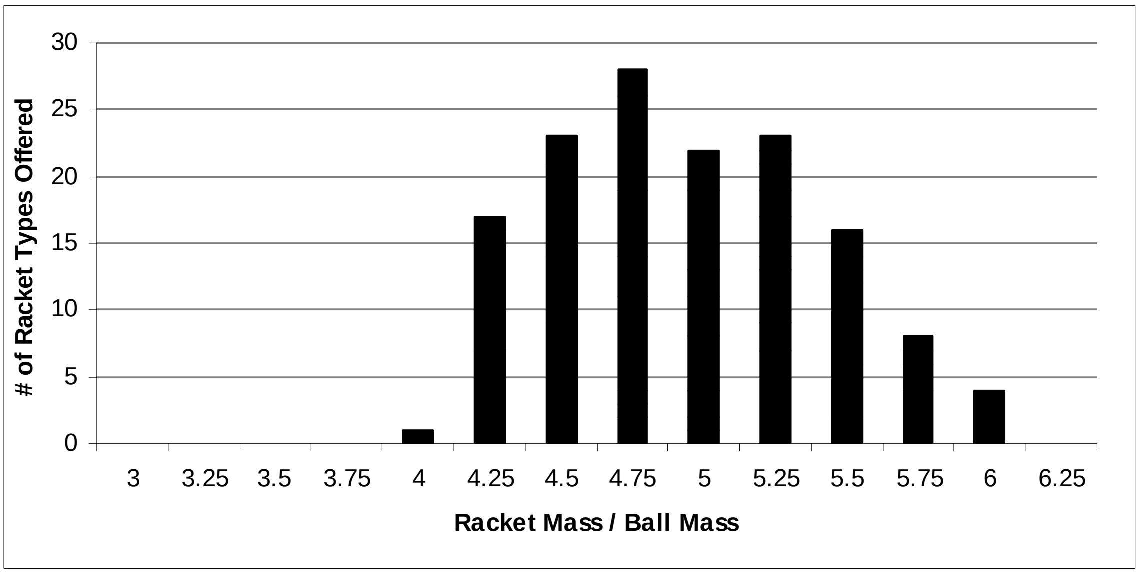
Since the early 1980s, there has been a trend of making tennis rackets lighter. The wooden rackets that were widely used until the early 80s weighed about 14 oz (7 times the weight of the ball). Currently, major tennis racket manufacturers offer graphite and composite rackets that are considerably lighter. The histogram below shows the number of racket models in given mass ranges offered by five racket manufacturers (Prince, Wilson, Head, Babolat and Yonex). The racket masses are plotted in units of tennis ball masses. The weight of a standard tennis ball is 2 oz. The manufacturers claim (and the popularity of the rackets bear out the claim) that lighter rackets result in increased shot speed for the average player. In this problem, we will investigate the advantages and disadvantages of lighter tennis rackets. (Photo by Chris Pawl.)



a.) We first investigate how racket mass affects the effort level needed to hit the ball with a certain speed. Assume that you are using a racket of mass M to hit a ball of mass m that is initially at rest. Further, assume that the collision is perfectly elastic. Using these assumptions, find a symbolic expression for the final speed of the ball (v_{shot}) in terms of the initial speed of the racket (v_{swing}).

b.) Using the result of part (a), plot the ratio of the initial racket kinetic energy to the final ball kinetic energy versus the ratio of the racket mass to the ball mass. In other words, plot $K_{\text{swing}}/K_{\text{shot}}$ versus M/m. The plot should include M/m values ranging from 0.25 up to 8.

Your plot from part (b) should show that the optimal kinetic energy transfer is achieved when the racket mass is the same as the ball's mass. Looking at the histogram of racket masses offered for sale, however, shows that commercial manufacturers do not currently offer rackets with masses less than about 4 times the ball's mass. In fact, tennis pros appear to favor rackets with masses 5-6 times the ball's mass. These heavier rackets are marketed as having better "feel" or "control". We can put these terms in a physics context. If you tried to use a racket with the same mass as the ball, then your racket would tend to suddenly stop when it contacted a ball at rest! This is certainly a jarring interruption of the swing. We can use this idea to examine the appropriate lower limit for tennis racket masses.

c.) Suppose that a ball was coming at you with a nonzero initial speed v_{ball}. Assuming that v_{ball} is exactly what is needed to stop your racket when you swing it at your normal speed v_{swing} (your racket's final speed will be zero) find a symbolic expression for the magnitude of v_{ball} in terms of v_{swing}.

d.) Using the expressions of parts (a) and (c) make a plot of $v_{\text{ball}}/v_{\text{shot}}$ as a function of the ratio M/m. Include M/m values from 1 to 8. Assuming that a normal tennis player should be able to return balls coming at them with v_{ball} at least as large as their own shot speed v_{shot} without having their racket come to rest, what is the minimum desired value of M/m?