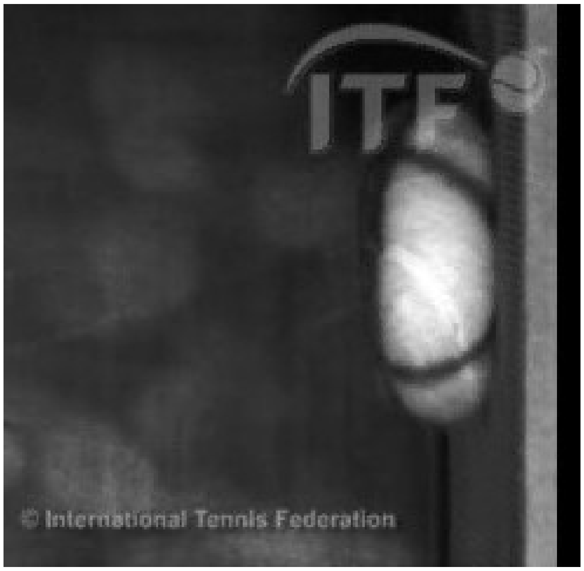
High-speed video allows scientists to examine aspects of collisions that happen too rapidly for the naked eye to resolve. The image below left shows the maximum deformation that occurs when a tennis ball allowed to freefall approximately 100 inches onto a wooden tabletop (image taken from video shot using the facilities of the Edgerton Center at MIT, with special thanks to Dr. James Bales). The image below right shows the maximum deformation that occurs when a tennis ball shot from a gas cannon impacts a concrete wall with an impact speed of about 50 mph (image taken from video available at the ITF Science & Technical Department website, http://www.itftennis.com/technical/, used with permission). (The black stripes on the ITF ball were added to aid in analysis of the video.)



One powerful feature of video analysis is that it gives data on two important aspects of the collision simultaneously. First, by examining the ball's compression in still images like those shown above, it is possible to estimate the distance over which the collision takes place. Second, by counting the frames during which the ball is in contact with the surface, it is possible to estimate the time duration of the collision. The fact that position and time are measured simultaneously also allows us to use the video to approximate the ball's incoming and outgoing speed.

This dual data collection allows us to compare two different ways to define average force. First, the concept of work and energy allows us to define a distance-averaged force for the collision by finding the change in kinetic energy and dividing by the distance over which the collision occurs. Second, the concept of impulse and momentum allows us to define a time-averaged force for the collision by finding the change in momentum and dividing by the time duration of the collision. In this problem, we will use data from the tennis ball collisions shown above to examine the difference between these two averages.

a.) For the 100 inch drop, the tennis ball was observed to approach the table top moving at a rate of 7.5 pixels per frame. The ball was in contact with the table for approximately 5 frames before (instantaneously) coming to rest at a peak compression of about 22 pixels (the ball's diameter was reduced from 156 pixels to 134 pixels, giving a compression of 22 pixels). Using this data, define the ratio of the distance-averaged force divided by the time-averaged force for the compression stage of the collision. Note that because you are asked for a ratio, you may work in units of pixels for distances and frames for time. Give a numerical answer, but show your work.

Even allowing for 10% errors, your answer to part (a.) should deviate from 1.0. The deviation is not surprising. The force provided by a tennis ball is not constant as the ball is deformed. Rather, the force increases as the ball is compressed (try it!). If the force was constant, we should have found that the time-averaged and distance-averaged forces were identical. For a non-constant force, the distance-averaged and time-averaged forces will generally differ.

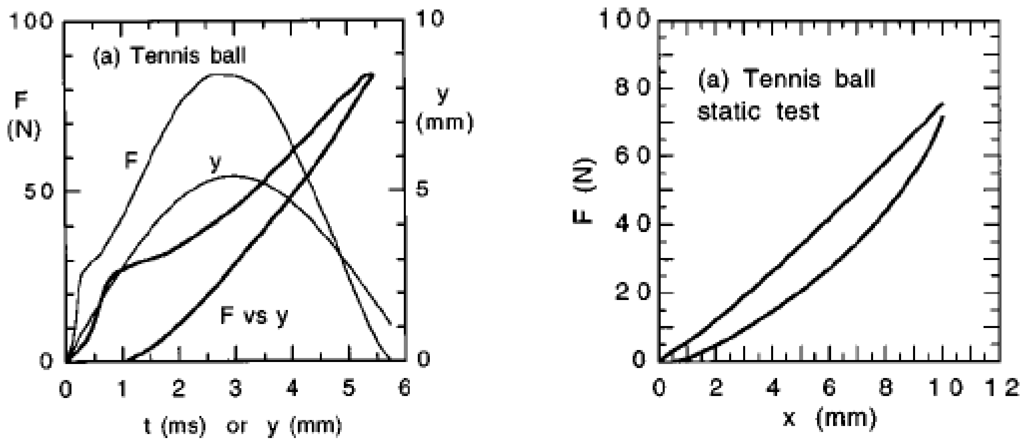
b.) As a simple model, suppose that the ball's force follows Hooke's law, $F = ky$ where k is a constant and y is the ball's compression. In that case, the ball would exhibit simple harmonic motion when impacting the tabletop, $y = y_{\max}\sin(\omega t)$. For this model, plot a graph of F vs. t and F vs. y for the compression phase of the collision.

c.) Using what you know about finding the average value of a graph, construct an argument using your graphs from part (b.) that explains why the distance-averaged force will be smaller than the time-averaged force for a collision that obeys Hooke's law.

d.) Find the exact ratio by evaluating the integral expressions for the average forces:

$$\frac{\overline{F}_{dist}}{\overline{F}_{time}} = \frac{\dfrac{1}{y_{\max}}\displaystyle\int_0^{y_{\max}} F(y)dy}{\dfrac{1}{t_{\max}}\displaystyle\int_0^{t_{\max}} F(t)dt}$$

e.) By performing a similar analysis for the rebounding part of the collision (the expansion of the ball as it leaves the table top) the ratio of the distance-averaged force to the time-averaged force was found to be approximately 0.67. This ratio deviates from the Hooke's law expectation by more than 10%. Below left is a plot showing the force and compression curve for a tennis ball impacting a surface at 2.95 m/s. By considering the F vs y curve and the F curve, construct an argument explaining why the force ratio during the rebound is less than the force ratio during the compression. It may help you to look at the static test result (obtained from a ball that was compressed manually while at rest), which is plotted below the collision graphs.



Above: Plots taken from "The bounce of a ball" by Rod Cross (*Am. J. Phys.* **67**, 222 (1999)). The left plot is generated by a tennis ball impacting a surface with an initial speed of 2.95 m/s. The plot displays three curves. The curve labeled "*F*" is the force of the collision as a function of time. The curve labeled "*y*" is the compression of the ball as a function of time. The thick curve labeled "*F* vs *y*" is the force as a function of compression. Note that the *F* vs *y* curves usually double back, because each compression value is attained once during the compression phase and once more during the expansion phase. The expansion phase will be the lower curve (can you support this conclusion using energy arguments?). The right plot is the force vs. compression curve for a ball compressed manually while at rest.

Challenge Problem: Download the ITF video "Bazooka close up with marked ball" dated 29 August 2008 at http://www.itftennis.com/technical/research/lab/projects/dynamics.asp. This is the video of the ball shot at 50 mph into a concrete wall. Analyze the video to determine the force ratios for this case. The frame rate of this video is 7104 frames per second. (Special thanks to Jamie Capel-Davies and the ITF Technical team for constructing a video suitable for analysis and passing along the parameters.)