INDUCTIVE INFLUENCE OF RELATED QUANTITATIVE AND CONCEPTUAL PROBLEMS

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Abstract

We address two questions: "Does conceptual understanding help students solve related quantitative problems?" and "Does working through a quantitative problem help students understand the concepts involved?" We approached these questions by splitting a class of approximately 100 students into two equally skillful groups, A and B. Using an tutorial electronic program led over 90% of students to the correct answer, we administer to group A, a conceptual problem followed by a related quantitative problem. To group B, we administered the same problems in reverse order. We found that working through a conceptual problem first had little effect on students' ability to solve a related quantitative problem. Conversely, our results suggest that working first through a quantitative problem improved students' ability to solve the subsequent conceptual problem.

Significance

The fact that a conceptual representation of the problem organizes the solution path by triggering the relevant formulas and groups of formulas is now well established. Although physics is a quantitative science, physics educators recognize that operating with formulas needs a background of qualitative knowledge (Jung, 1993). This seems to imply that the concepts should be taught first, as they often are using Peer Instruction (Mazur, 1997).

Surprisingly, the present study suggests that the reverse order works better. We have found that the students were successful in solving a conceptual problem when they had previously used those concepts in working through a related quantitative problem. On the other hand, we found that solving a conceptual problem first did not make a significant improvement a student ability to solve a related quantitative problem. This finding of cause-effect relationship is likely to be a source a discussion among researchers interested in epistemological issues (or assumptions) and how they influence learning.

Theoretical Underpinnings

Physics education researchers have documented the dramatic gap between conceptual learning and quantitative learning over the past few decades (Frederiksen, 1984; Tuma & Reif, 1980). Diagnostic tests like the Force Concept Inventory and the Mechanics Baseline reveal deep conceptual misunderstanding of seemingly simple physical concepts. Interactive teaching techniques such as Peer Instruction (Mazur,

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1997) and context rich problems (Anderson et al., 1992) can dramatically improve student scores on such tests. However, the corresponding increase in students' abilities on standard quantitative problems is not proportional. This suggests that the students are not learning an integrated approach to problem solving. The present study is designed to probe the relationship between conceptual learning and quantitative learning.

Quantitative problems dominate physics examinations. Cognitive science, however, has demonstrated that quantitative problem solving needs a background of qualitative (conceptual) knowledge. "Without it, we may have formulas but no physics" says Jung (1993, p. 44). At the same time, Jung (1993) also affirms that the problem-solving behavior can be improved when treated by a more explicit procedure, that of discussing the underlying general schemas. In physics, relations between variables represent schemas, as do concepts like conservation of momentum.

The qualitative schemas are seen as important first steps in the solution process. Analysis of learning difficulties and misconceptions leads to obvious attempts to improve teaching procedures by making the schemas more explicit. If we understand explicit procedure as the fact of solving the problem using quantitative reasoning, then would the explicit procedure help students understand the conceptual underpinnings of a problem? This controversial concept suggested by the findings in the present research, would challenge researchers such as Neto & Valente (1997) who suggest that teachers must put greater focus in more qualitative approach to problem solving.

Students must build mental models, and these are built by observation as Edward Redish (1994) has suggested. Perhaps most students learn general concepts from particular examples, as Diana Laurillard (1993) has also suggested in *Rethinking University Teaching,* rather than the other way around. If these ideas were applicable here, the integration of concepts into the solution of quantitative problems would be educationally fruitful. Our data suggest that this may be the preferred approach.

Procedure

The current study was undertaken during the spring term of 2001 in the required Introductory Newtonian Mechanics with Calculus course at MIT. Most students in the spring term had failed to get a grade of C or better in a previous attempt and were taking the course second time. Problem pairs were administered using CyberTutor, a web-based tutorial program. CyberTutor uses a Socratic method offering students help upon request in the form of hints and simpler subproblems, spontaneous warnings and suggestions when wrong answers are given. If the student exhausts the available hints they can request the solution to the problem. Using CyberTutor, typically about 90% of the students worked their way through to the solution of each problem, the remaining 10% requested the solution or gave up. CyberTutor keeps a record of the number of hints requested (h) right (r) and wrong (w) answers submitted, and solutions (s) requested. For purposes of the present research we use a single ad hoc measure of the difficulty that a student has in working a particular problem and it is based on reasonable weighting of these data.

$$Difficulty : D = Log \frac{1 + r + w + 3*h + 9*s}{1 + r}$$

The difficulty is 0 if only right answers are given and rises to around 3 for students requesting all the hints and solutions with no right answers. Values around 1 appear to be optimal in an educational research frame.

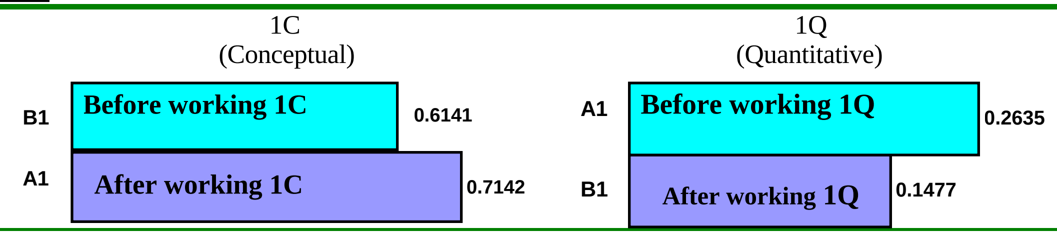
The A and B groups of the class were balanced according to which math course the students were taking and the score on assignments given prior this study. Equal numbers of students taking a more advanced math course (than the standard spring semester calculus on multi-variable functions), equal numbers were taking the first semester calculus course (most for the second time), and found the problems on the first two assignments on CyberTutor (http://CyberTutor.mit.edu) of equal difficulty. In terms of difficulty the two groups reminded well balanced over the entire term, never differing by more than 4%.

Conceptual and Quantitative Pairs

The first pair of problems was concerning two body collisions in one dimension. Problem 1C is largely conceptual and 1Q is quantitative.

<u>Problem 1C [Conceptual]. Bullet Embedding in a Block</u>
This is a conceptual problem that possesses 3 parts concerning elasticity, kinetic energy, and speed of the bullet and block system respectively.
<u>Problem 1Q [Quantitative/numerical]. One-Dimensional Inelastic Collision</u>
This problem describes a block that collides with another block and asks the student for both analytical and quantitative responses about elasticity, velocity and kinetic energy.
(See appendix for the complete problem)

Table 1
<u>Measured Difficulties for Problems 1C and 1Q Depending on the order in Which They Were Taken</u>



N	A1=31 students, B1= 47 students	
ΔD	−0.1158	+0.0996
p-value[2]	0.046	0.56

The p-value is the probability that the observed discrepancy between the two sample means is due to chance. Under the usual criteria that p-value < 0.05 is significant, we find that the students' experience with the quantitative problem was a significant help with their performance on the conceptual problem (it decreased the difficulty by ΔD = -.1158) (Table 1). The decrease in difficulty was the result of an average of 45% fewer wrong answers, w, 71% fewer requests for hints, h, and 0% fewer requests for correct solutions, s. If anything, students found the quantitative problem 1Q of *greater* difficulty (ΔD = +.0996) where they had first worked through the conceptual problem 1C.

<u>Other Conceptual/Quantitative Pairs</u>

The results from other conceptual and quantitative problem pairs are not as significant as the previous example. Due to growing (over the term) student self-selection of the order in which the problems were taken, a large percentage of the class ended up

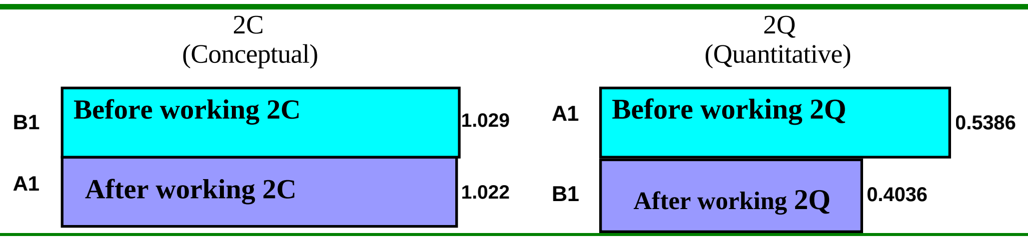
[2] The p-value in the table is the double-sided value found using Student's t-test (Horowitz, 1974). The more conservative two-sided p-value is appropriate when sign of the effect is not known a priori.

taking pairs in one order with very few taking them in the reverse order. One such example involved two problems dealing with contact forces and friction

> <u>*Prob. 2C. [Conceptual] Contact Forces Explained*</u>
>
> *This is a conceptual problem which begins by describing the general nature of contact forces and friction, then poses three multiple-choice questions for the student to answer.*
>
> <u>*Prob. 2Q. [Quantitative] Friction of a man on a drawbridge*</u>
>
> *This problem describes a mass sliding with friction on an incline and asks the student for both quantitative and analytic responses.*
>
> *(See appendix for the complete problem)*

The results for this pair of problems are given in Table 2. If we view this problem as a test of the hypothesis that the quantitative problem helped solve the conceptual one, the appropriate p-value used is the (single sided) value $p=0.042$, again a significant confirmation of this hypothesis.

Table 2
<u>Measured Difficulties for Problems 2C and 2Q depending on the Order in which They Were Taken</u>.



N	A1=70 students, B1= 17 students	
ΔD	−0.135	−0.007
p-value	0.0843	0.9717

A third conceptual-quantitative comparison was performed in two-dimensional kinematics. A conceptual problem involving the direction of acceleration of a car going around an irregular racetrack at changing speeds was placed before and after a *pair* of quantitative problems: taking the derivative of a time-dependent x-y position, and a similar problem involving circular motion. All measured inductive influences were small and statistically insignificant in this example.

Findings

Two findings emerge from the present study:

1) Comparing three conceptual problems with four quantitative ones, we found no evidence of inductive influence of a previously administered conceptual problem on a subsequent quantitative problem on the same topic. In contrast to this result, similar studies of the inductive influence of a numerical problem and a similar quantitative one, as well as the inductive influence of two closely related analytic problems show frequent reductions of difficulty over 50% with strongly statistically significant p-values.This finding challenges researchers such as Neto & Valente (1997) who suggest that teachers must put greater focus on qualitative approaches to problem solving.

2) We found statistically significant examples of inductive influence in which working a *quantitative* problem through to its solution reduced the students' difficulty with a subsequent *conceptual* problem. This is a preliminary result of strong statistical

significance. Additional data to probe whether this is due to as yet unidentified systematic feature of our procedure are currently under way.

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Appendix

Here is a pair of problems concerning two body collisions in one dimension. Problem 1C is largely conceptual and 1Q is quantitative.

Problem 1C [Conceptual]. Bullet Embedding in a Block

A bullet of mass m is fired horizontally with speed v_0, aimed at a block of mass M resting on a frictionless table. It hits the block, and becomes completely embedded. The block and bullet then move at speed v_f.

Part A. Which of the following best describes this collision?

- perfectly elastic.
- partially inelastic.
- perfectly inelastic.
- none of the above.

Part B. Which of the following quantities, if any, are conserved during this collision?

- kinetic energy.
- kinetic energy and momentum.
- momentum.
- none of the above.

Part C. What is the speed of the block and bullet system after the collision in terms of v_0, m, and M?

Problem 1Q [Quantitative/numerical]. One-Dimensional Inelastic Collision

Block 1, of mass 100 g, moves along a frictionless air-track with speed 0.2 m/s. It collides with block 2, which was initially at rest. Block 2 has mass 200 g. The blocks stick together and move as one after the collision.

Part A. Find the total initial momentum p of the two-block system in units of kg m/s.

Part B. Find the final velocity v of the two blocks in m/s.

Part C. What is the change in the system's kinetic energy due to the collision? Express your answer in Joules.

A total of 78 students worked both problems. The 31 students who took problem 1C first followed by problem 1Q are group A1; the remaining 47 students in-group B1, took the problems in reverse order. The average measured difficulty, D, for each group of students for each problem and the change in the difficulty after experience with the other problem is listed in Table 1 where D represents is the average difficulty per group.

Here is a second pair of problems concerning Forces. Problem 2C is largely conceptual and 2Q is quantitative:

Prob. 2C. [Conceptual] Contact Forces Explained

Two solid objects cannot occupy the same space at the same time. Indeed, when the objects touch, they exert repulsive forces on each other, as well as frictional forces which resist their slipping relative to each other. These contact forces come from a complex interplay between the electrostatic forces between the electrons and ions in the objects and the laws of quantum mechanics. As two surfaces are pushed together these forces increase exponentially over an atomic distance scale, easily becoming strong enough to distort the bulk material in the objects if they approach too close. In everyday experience, contact forces are limited by the deformation or acceleration of the objects, rather than the limitations of the fundamental interatomic forces. Hence their magnitude is determined by the requirement that they, together with any other forces on the contacting bodies, produce the observed acceleration of the bodies.

Normal and Friction forces

Two types of contact forces operate in typical mechanics problems, the normal and frictional forces, usually designated by N and Ff (or Ffric, or something similar) respectively. These are the components of the overall contact force perpendicular and parallel to the plane of contact.

Kinetic Friction when surfaces slide

When one surface is sliding past the other, experiments show three things about the friction force:

1. the frictional force opposes the relative motion at the point of contact,
2. Ff is proportional to the normal force, and
3. the ratio of the magnitude of the frictional force to the normal force is fairly constant over a wide range of speeds.

The constant of proportionality is called the kinetic coefficient of friction, often designated μk. As long as the sliding continues, the frictional force is then

Ff = μk N (valid when the surfaces slide by each other).

Static Friction when surfaces stick

When there is no relative motion of the surfaces, the frictional force can assume any value from zero up to a maximum μs N, where μs is the static coefficient of friction. Invariably, μs is larger than μk, in agreement with the observation that once something breaks loose and starts to slide, it often accelerates.

The frictional force for surfaces with no relative motion is therefore

$Ff \leq \mu_s N$ (valid when the contacting surfaces have no relative motion).

The actual magnitude and direction of the static friction force is such that it (together with other forces on the object) causes the object to remain motionless with respect to the contacting surface as long as the static friction force required does not exceed μs N.

(In the above, the symbol μ is the Greek letter ``mu'', pronounced with a long ``U''; like most symbols, μ can appear in many different fonts. The symbol μ is also the prefix for ``micro'', or ``one-millionth''; a common usage is 1 μm = 10-6 m, and 1μ m is known as a ``micron''.)

Answer the following multiple-choice questions; if you need a hint, reread the description above.

A. When two objects slide by one another, the frictional force between them can best be described

- = μk N
- < μk N
- is determined by other forces on the objects

B. When two objects are in contact with no relative motion, the frictional force between them

- = μs N
- < μs N
- is determined by other forces on the objects
- none of the above

C. When a board with a box on it is slowly tilted to larger and larger angle, common experience shows that box will at some point ``break loose'' and start to slide down the board with increasing speed unless angle is quickly reduced. The most general explanation for this is

- μk is less than μs
- adhesive or dirt initially sticks the box to the board
- the force that starts the box sliding continues to act

· - sliding reduces the normal force which reduces the frictional force

Problem 2Q [Quantitative]: Friction of a man on a drawbridge

A person is standing on one leg on a drawbridge that is about to open. For all the questions, we can assume the bridge is a perfectly flat surface and lacks the curvature characteristic of most bridges. Two constants that you need to know are:

μs, the coefficient of static friction between the drawbridge and the person's foot, and μk, the coefficient of kinetic friction. Use Fn to represent the normal force exterted on the man by the bridge. In your answers, enter mu_k for μk and mu_s for μs.

A. Before the drawbridge starts to open, it is perfectly level with the ground. The person is standing still on one leg. What is the x-component of the friction force, Ff? Answer in terms of any or all of the following variables: Fn, mu_s, and/or mu_k.

Ff =

B. The drawbridge then starts to rise and the person continues to stand on one leg. The drawbridge stops just at the point where the person is on the verge of slipping. What is the magnitude of the frictional force now? Answer in terms Fn, mu_s, and/or mu_k.

Ff =

C. Then, due to the bridge being old and poorly designed (the designer didn't take 8.01), the bridge falls a little bit and then jerks. This causes the person to start to slide down the bridge at a constant speed. What is the magnitude of the frictional force now? Answer in terms of Fn, mu_s, and/or mu_k.

Ff =

D. The bridge starts to come back down again. The person stops sliding. As the bridge is almost down, another of the mechanic's blunders shows itself. The bridge never makes it all the way down, rather it stops half a foot short. This half a foot corresponds to the angle theta << 1°. (see the diagram, which has the angle exaggerated) What is the force of friction now? Answer in terms of theta, Fn, Fg, mu_s, and/or mu_k.

Ff =

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