Mining the Internet for Data I: Sports Equipment

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Problems using typical numbers for sports equipment parameters such as: “A 0.285 kg tennis racket strikes a 0.058 kg tennis ball…” are common in introductory physics. The numbers are usually reasonable, but presenting only one set of representative numbers can often overlook interesting features of the real world. Continuing with the example above, tennis ball masses are tightly constrained by the International Tennis Federation (ITF) to range between 56.0 g and 59.4 g, but the rules do not restrict the mass of tennis rackets[1]. Instead, physics plays a role in fixing the preferred tennis racket mass. In this article, we give an example of how internet research using the readily available commercial websites of sports equipment manufacturers can enrich introductory physics problems and spark interesting follow-up questions.

Fig. 1: A typical 1970’s wooden racket and a typical modern composite racket (photo by Chris Pawl).

Tennis rackets have undergone a major evolution in the past 30 years. Until the early 1980s most players used wooden rackets which were constrained by structural limitations to weigh about 14 oz (or about 7 times the weight of the 2 oz ball). By the mid 1980s, however, carbon fiber and fiberglass rackets, which could be made larger, stronger, and lighter than wooden rackets, had come to dominate the game (Fig. 1). Since that time, racket manufacturers have competed to market lighter rackets. Fig. 2 is a histogram of the adult racket types marketed by five major racket manufacturers constructed using data obtained from their commercial websites. The histogram clearly shows that the days of 14 oz rackets are over, but interestingly, it also displays a sharp cutoff at racket masses less than about four times the ball’s mass. In this article, we use basic collision analysis to explore the implications of this cutoff. The problem is a rich one, yielding unexpectedly simple results, but also suggesting a wide range of possible complications for further exploration.

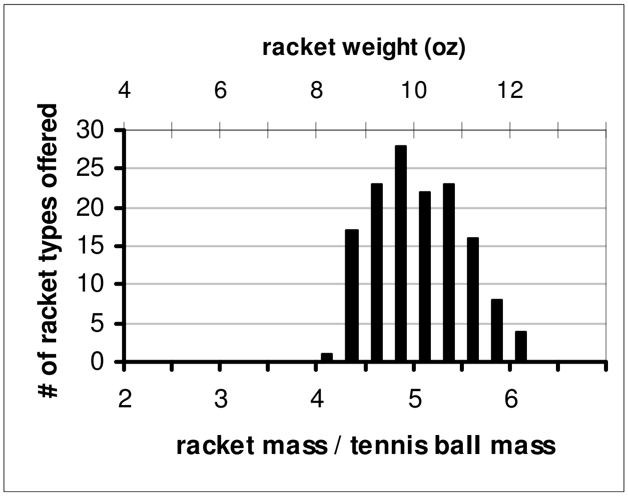

Fig.2: Histogram of adult tennis racket types marketed by five major manufacturers (Prince, Wilson, Head, Babolat and Yonex).

It is possible for introductory physics students to analyze the data of Fig. 2 in a simple but powerful fashion. We will ignore the complications implied by the fact that the racket is actually pivoted rather than translated and that it may not contact the ball at its center of mass (see e.g. Refs. 2 and 3 for a more detailed analysis). We will begin by using the simplest possible model of the tennis stroke: *a one-dimensional elastic collision between two point masses*. It will prove useful to define the basic element of our analysis to be the shot speed v_{shot} that a particular player can give to a ball that is struck at rest. A basic elastic collision analysis leads to the following formula for v_{shot} in terms of v_{swing}, the speed of the player's racket before impact with the ball:

$$v_{\text{shot}} = \frac{2M}{m+M} v_{\text{swing}} \qquad (1)$$

where m is the ball's mass and M is the racket's mass. This shot speed is a useful quantity because people tend to play against competitors of about their own ability. Thus, we propose the following hypothesis: *a useful tennis racket should have a mass that allows a player to handle a shot hit by an opponent of the same ability.*

We will now attempt to use the data of Fig. 2 to explore what is meant by "handle". We will focus on racket recoil. Racket recoil is very much dependent on racket mass. For example, a racket with $M = m$ is clearly unacceptable in our simple model, since a collision with a moving ball during play would cause the racket to recoil backward with the speed of the incoming ball. This suggests a first definition of "handle". We will begin with the definition: *a racket has "handled" a shot if it swings through the collision without recoiling*. Suppose a player strikes a ball that is approaching their racket with a nonzero initial speed. If the player swings the racket with their typical speed v_{swing}, another elastic collision analysis can be made to determine the value of the ball's velocity before the collision (v_{ball}) that will completely stop the racket (the racket's final speed is zero). The requisite ball velocity (v_{stop}) satisfies:

$$\left|v_{\text{stop}}\right| = \left|\frac{M-m}{2m} v_{\text{swing}}\right|. \qquad (2)$$

We can now test our hypothesis of equal opponents with our initial definition of "handle" by assuming that the player's racket should be capable of swinging through a collision with a ball which is incident at the player's own shot speed (v_{shot}) without tending to recoil in the opposite direction. By setting Eq. (1) equal to Eq. (2), we find that the minimum acceptable mass ratio is $M/m = 4.24$. Fig. 3 illustrates the dependence of v_{stop} on the mass ratio ($M/m = 4.24$ is the point at which this curve has a value of 1). The agreement between our approach and the histogram of racket masses is excellent. In fact, the agreement is so good that it makes sense to check the predictions of this approach for another sport.

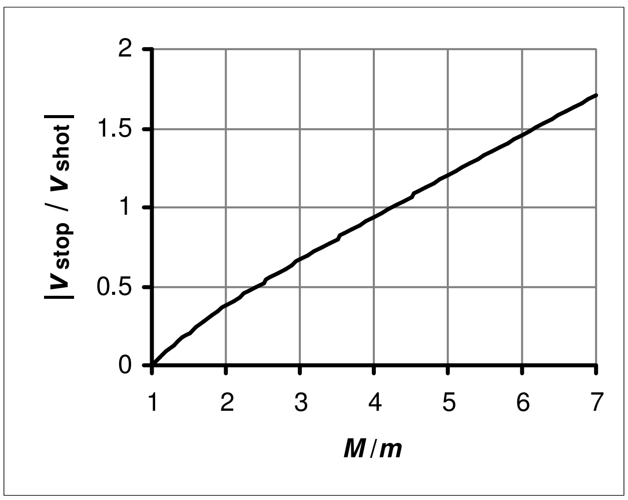

Fig 3: The ratio of the initial ball speed required to stop a player's swinging racket to the player's typical shot speed when hitting a ball at rest.

Baseball and softball provide a reasonable test of the predictions of our approach. Advances in technology have allowed bat manufacturers to market lighter bats in a manner analogous to the evolution of tennis rackets. Histograms of commercial non-wood baseball and softball bat masses are shown in Figs. 4 and 5. Interestingly, adult baseball and softball bats show substantial deviation from the limit $M/m > 4.24$. Thus, we must conclude either that our hypothesis of equal opponents, our definition of "handling" a hit, or the model itself is in error. Many reasons for any (or all) of these options might be considered. We suggest three possibilities here. (Note that we have already used the lower limit on the ball masses in creating Figs. 4 and 5 so that any bias is in our favor.)

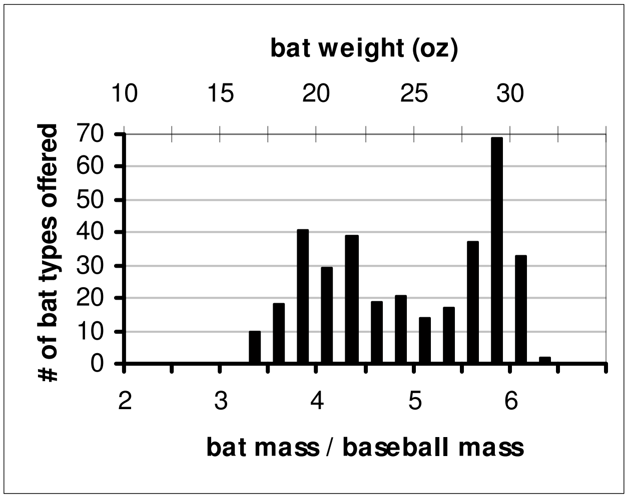

Fig. 4: Histogram of adult non-wood baseball bat types marketed by five major manufacturers (deMarini, Easton, Louisville Slugger, Worth and Rawlings). The peak at *M/m* ~ 5.75 is a result of high school and NCAA rules limiting the length – mass differential of legal bats (an interesting example of subtracting two quantities with different units).

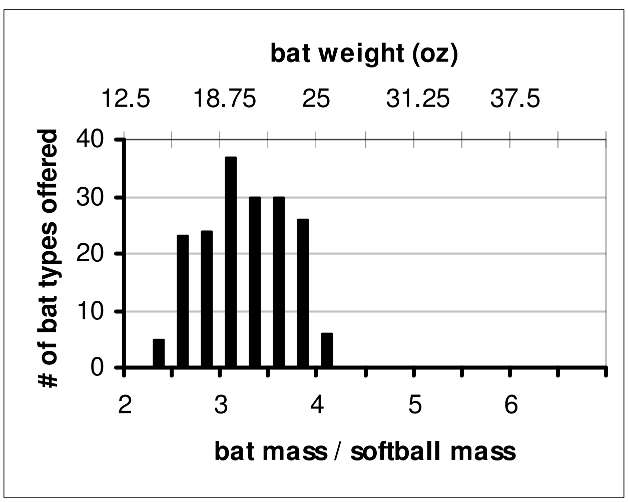

Fig. 5: Histogram of adult softball bat types marketed by five major manufacturers (deMarini, Easton, Louisville Slugger, Worth and Mizuno).

First, it might be supposed that our hypothesis of equal opponents is not relevant to baseball and softball. Perhaps the pitcher throws the ball with a speed substantially less than that with which the ball leaves the bat when struck at rest. Some research into typical numbers, however, shows that the assumption that players must be ready to handle $v_{\text{ball}} \sim v_{\text{shot}}$ is reasonable for both baseball[4] and fast-pitch softball.

Second, it might be that the model of a one-dimensional collision is in error because rotational effects play a larger role in the bat-ball collision than they do the racket-ball collision. A significant literature exists addressing the nuances of the baseball-bat collision[5,6]. Advanced students could be invited to investigate these details.

We will focus in this article on a third possibility, suitable for analysis in an introductory class. The official rules of tennis[1], NCAA baseball[7] and NCAA softball[8] each place limits on the coefficient of restitution (COR) of legal balls. The allowed ranges are 0.73-0.76, 0.555 (max) and 0.47 (max) respectively. These coefficients are specified with respect to impacts with surfaces rather than the racket or bat. (Existing literature suggests a range of possible COR values of 0.6-0.9 for the tennis ball-racket collision[2,9] and of 0.45-0.6 for the baseball-bat collision[4,10].) A COR less than 1.0 implies that the collision is not perfectly elastic. *To allow for a COR, we must adjust our model by replacing the elastic collision requirement that kinetic energy be conserved with the constraint equation:*

$$e(v_{\text{swing, i}} - v_{\text{ball, i}}) = v_{\text{ball, f}} - v_{\text{swing, f}} \qquad (3)$$

where e is the COR and the subscripts "i" and "f" denote the before-collision values and the after-collision values respectively. This modification alters the form of Eqs. (1) and (2). They are replaced by:

$$v_{\text{shot}} = \frac{(1+e)M}{m+M} v_{\text{swing}} \qquad (1')$$

$$|v_{\text{stop}}| = \left| \frac{M - em}{(1+e)m} v_{\text{swing}} \right|. \qquad (2')$$

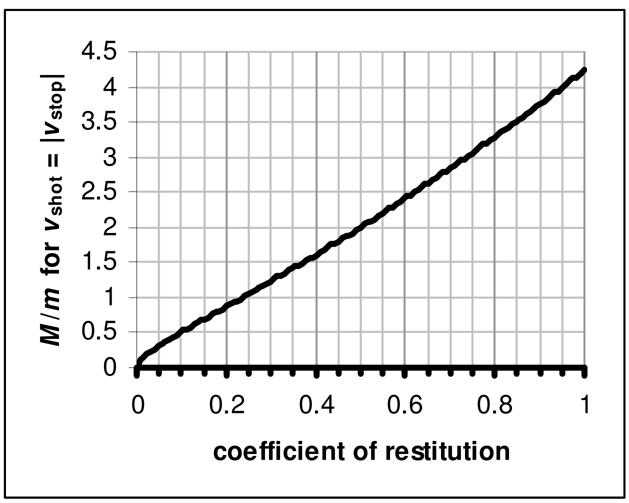

Fig. 6: The mass ratio such that a ball incident with the player's shot speed (as defined by Eq. (1')) will bring the player's racket to rest during the collision as a function of the coefficient of restitution.

Fig. 6 plots the value of the ratio M/m which results in $v_{\text{shot}} = |v_{\text{stop}}|$ as a function of the COR. This plot shows that accounting for the COR can explain the most prominent trend in the lower cutoff of M/m values observed in the sequence of Figs. 2, 4 and 5. Because softballs have a substantially lower COR than tennis balls, players are willing to use bats with M/m significantly lower than that of tennis rackets. Fig. 6 brings up new questions as well. By our initial definition, all three sports now exhibit a racket or bat mass cushion. The major manufacturers studied here do not market rackets or bats to adults that are light enough to reach the limit $v_{shot} = |v_{stop}|$. This leads us to re-examine our definition of what it means to "handle" a shot.

A possible redefinition that builds on what we have done is to assume that a player's racket should retain some nonzero fraction of the swing speed after the collision. If we continue to realize our hypothesis by assuming that the ball impacts the racket with the same speed as the player's shot speed from rest (i.e. we assume $|v_{ball}| = v_{shot}$ as defined in Eq. (1')), the racket's retained speed, $v_{retained}$, is given by:

$$v_{retained} = \frac{M^2 - (3e + e^2)Mm - em^2}{(M + m)^2} v_{swing} . \qquad (4)$$

Fig. 7 plots the value of the ratio *M*/*m* required to retain a certain fraction of the swing speed as a function of the COR. Looking at this plot shows that our simple approach gives a good description of the relative mass cutoffs if we assume that the retained swing speed is around 15-20%. For example, to retain 16% of the swing speed, Eq. (4) suggests M/m > 4.0, 3.0 and 2.5 for COR values of 0.75, 0.55 and 0.45, respectively.

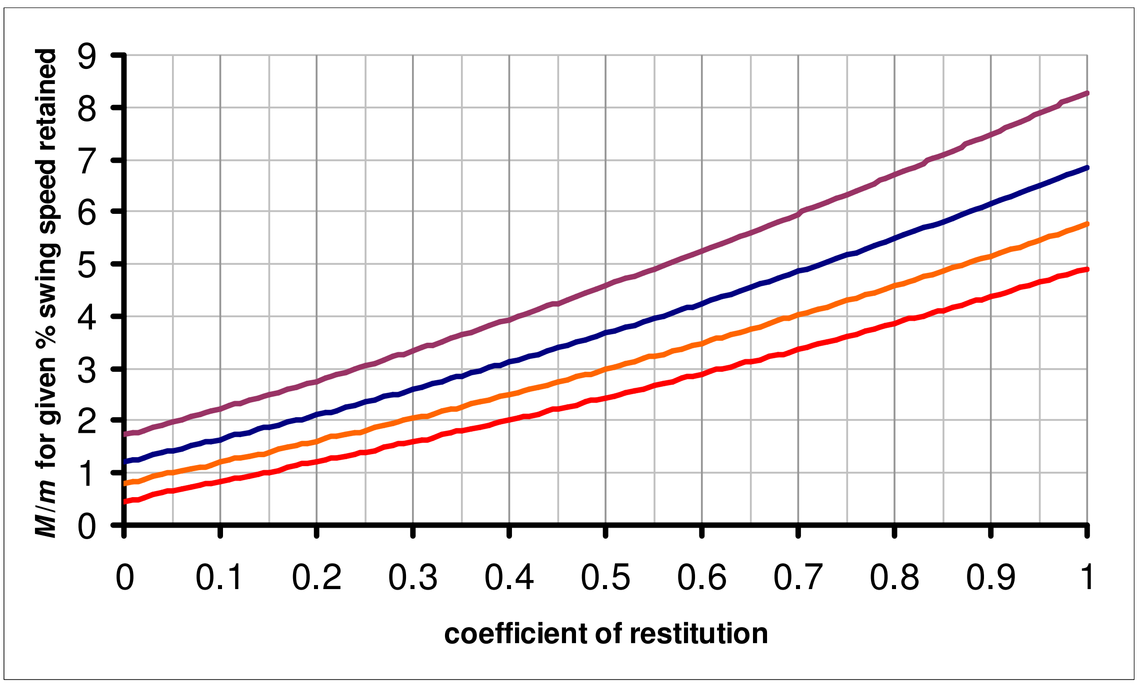

Fig 7: Curves showing *M*/*m* ratio required to retain 10% (red, bottom) through 40% (purple, top) of the initial swing speed in steps of 10% as a function of the coefficient of restitution.

Our approach has now reached what seems to be a reasonable description of the observed cutoffs, and in the process we have refined our definition of "handling" a shot to state that a minimum of about 15-20% of the racket or bat speed should be retained after the collision. Students might be interested in further investigation. One possible follow-up project is to collect data on the rackets and bats that are marketed for youth and compare their *M*/*m* values to the range found for adult equipment. Another is to investigate sports such as golf, where the ball is at rest before contact. Advanced students could examine the impact of rotational effects, construct experiments to test the model, or explore the literature devoted to predicting the optimal bat mass from analysis of swing speed[11].

In this article we have described how gathering a statistically sizeable data sample from the internet can enrich a typical introductory physics problem. In this case, the data can be used to illustrate steps in the development of a theoretical model. The analysis of the data also suggests follow-up projects that are at a reasonable level for introductory physics. For more advanced students, a project like the one described can serve as a springboard to a literature review. A freely-available problem based on the material presented here along with other examples of using data to enrich standard physics problems can be found at our website[12].

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