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Effectiveness of Tutorials in Web-based Physics Tutor

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Abstract

We study two pedagogies using myCyberTutor, a web-based homework tutor. Two groups, tutorial-first (TF) and problem-first (PF), taking introductory mechanics solve a tutorial and a related problem pair. The TF group solves the tutorial problem before solving the related problem(s) while the PF group solves the related problem first. The TF group has significantly less difficulty on the related problem(s) and completes them more quickly than the PF group. The PF group shows a smaller reduction in the difficulty they have with the self assessment questions in the tutorial. The reduction in difficulty of the TF group on the related problem is twice as much per unit of time spent on the tutorial as is the corresponding reduction in difficulty experienced by the PF group. We conclude that schema acquisition is better facilitated by the tutorial-first approach.

Computer technology in general and the internet in particular have facilitated and even motivated the development of interactive tutors for individual students. Tutoring of students in such an environment allows the collection of data showing the degree of difficulty that such students experience in solving a particular problem and the time to solution. This provides an opportunity, as shown in this paper, to assess the amount learned per unit time from prior tutoring as measured by the reduction in difficulty on a subsequent problem.

One goal of a computer- or a web-based tutor is to help transform novices into experts. In such a transformation the construction of necessary problem-solving schemas play an essential role. We take problem-solving schemas to mean mental representations which are useful in organizing information in meaningful ways that facilitate understanding of concepts based on past experiences with particular types of problems. (Kahney, 1993; Sternberg & Ben-Zeev, 2001). The role of schema in facilitating transfer of knowledge among problems is based on the assumption that a schema makes it easier to access information relevant to the problem in hand, the relevant schema been built through previous experience (Gick & Holyoak, 1983; Kahney, 1993).

In this paper we evaluate two strategies for teaching schema – tutorial-first and problem-first - using the web-based Socratic “tutor” – myCyberTutor (http://www.mycybertutor.com). In the tutorial-first (TF) pedagogy students first solve a tutorial problem before solving a problem that is related to the tutorial in concepts and methods. In the problem-first (PF) pedagogy students first solve a problem followed by a tutorial problem which discusses the concepts and methods employed in the related problem. The tutorial-first approach may be considered closest to deductive reasoning where students solve problems starting from previously learned general principles that are meant to be true for all such cases. Traditional teaching is generally deductive and is encouraged by instructional systems development models (Gagné, Briggs, & Wager, 1992; Reigeluth, 1983). In contrast, the problem-first approach may include elements of inductive learning where learners construct their own concepts and rules based on their interpretation of particular cases (Gagné, Briggs, & Wager, 1992; Anderson, 2000). One example is discovery learning. We hypothesize that the tutorial-first pedagogy facilitates the acquisition of schema which involve representations of optimal abstraction (Kahney, 1993). Our assessment is not based on the standard methodology of whether or not a student has submitted the correct solution (Reif & Scott, 1999), but rather on a more elaborate set of variables which allow us to assess students more reliably (Pritchard & Morote, 2002). As such, this paper illustrates the power of assessment using a web-based tutor to collect data which are unavailable in standard assessments.

General pedagogy of myCyberTutor

The overall pedagogical approach adopted in myCyberTutor is closest to mastery learning (Bloom, 1968, 1976) where time to learn is allowed to vary in order to make students achieve mastery (obtain a score of 80-100% on an assessment) of a given topic. This is the reverse of most in-school instruction where the time is fixed and only the most skillful students master the material. Mastery learning is implemented within a Socratic dialogue where students are provided with hints and simpler sub-problems upon request, and given specific criticism when incorrect answers are proposed. With myCyberTutor, typically about 60% of the students provide the correct answer on a given question the first try and over 90% obtain the solution after receiving feedback and hints.

The hints and sub-problems outline and then detail a straightforward solution: typically the hints would provide students with the necessary declarative knowledge, while the sub-problems would evaluate the important procedural knowledge to solve the problems in a straightforward way. Responses are available to common student errors on each question. In addition, follow-up comments are frequently given to highlight important features or implications of an answer that has just been obtained. The follow-ups are designed to foster active engagement of the student (Redish, Saul, & Steinberg, 1997; Sokoloff & Thornton, 1997). If the students exhaust the available hints they can request the solution to a problem or a sub-problem.

Tutorial and related problems

The myCyberTutor problem library contains both tutorial and self-tutoring problems. Tutorial problems in myCyberTutor are best characterized as carefully planned and sequenced instruction. The tutorial problems are called “skill builders” in Mastering Physics (www.masteringphysics.com), a commercial version of myCyberTutor. These tutorials have the following features:

- They begin by stating a particular “learning goal.”
- They often include expository text similar to what might be found in a textbook.
- They ask questions to elicit the conditions under which the concept is applicable.
- If a formula is involved, they ask questions to elicit what the various terms in the formula mean.
- They walk the student step-by-step through one or two applications of the concept.

Some self-tutoring problems in myCyberTutor are designed for practice in applying or extending a concept, a formula, or a procedure assumed to be familiar from previous class discussions, by reading texts, or by working tutorials. Some challenge the student to apply a familiar concept in significantly more complicated situations, including situations that require applying several distinct concepts at once. The self-tutorial problems in this paper are of the former type. Concepts, formulae, and procedures necessary to solve the problems are generally available only through the hints and sub-problems, but not in the problem as initially presented (Mayer, 1983).

In this study each tutorial problem is paired with a self-tutorial problem that involves the same concepts. We call these self-tutorial problems the "related problems." When students solve a related problem without a previous tutorial they may be constructing knowledge by themselves and/or relying on previous knowledge. The related problems have the following features:

- They challenge the student to answer questions for which the related concept is applicable.
- They indicate the necessary concept or the formula and provide a step-by-step walk-through on how to apply it only in the hints and sub-problems.
- They immediately point out to the student the failures of many incorrect responses.

The present study involves three tutorial problems (T1, T2, and T3) and their related problems (R1A, R1B, R2, and R3). The problems can be categorized according to their learning goal.

1. First group: Learning a procedure

T1: Torque - z component

R1A: Torque practice-1

R1B: Torque practice-2

Problem T1 explains both the moment arm and the tangential force methods to find torque. The related problems R1A and R1B are designed to practice the procedures learned in T1 (see Appendix A). All problems in this category require algebraic or quantitative responses.

2. Second group: Understanding a concept

T2: Newton's 3rd law presented and discussed

R2: A book on a table – identify 3rd law force pairs

Problem T2 is conceptual with the goal of helping students understand a physical law (Newton's 3rd law). Problem R2 is also largely conceptual, requiring "fill in the blank" answers (see Appendix B).

3. Third group: Expressing a formula

T3: Harmonic oscillator kinematics using trigonometric functions and calculus

R3: Graphical position, velocity, acceleration of oscillator

Problems T3 and R3 take into account the previous findings that show that students often connect velocity with changing position of moving objects (Trowbridge & McDermott, 1980) and confuse velocity and acceleration or create analogies between them (Trowbridge & McDermott, 1981; Jimoyiannis & Komis, 2001). Problem T3 is a tutorial to help students express a formula as a function of other variables and is both conceptual and quantitative in nature while its related problem R3 is largely quantitative (see Appendix C).

In this paper, we study the efficiency of two pedagogical presentations of these web-based problems: In the tutorial-first (problem-first) approach, the students solved a tutorial (related) problem-first, and a related (tutorial) problem second. Our study then measures the amount by which the experience on a given tutorial (related) problem makes it easier for students to solve the following related (tutorial) problem(s).

Design

The current study was undertaken during the spring term of 2002 in the required “Introductory Newtonian Mechanics with Calculus” course at the Massachusetts Institute of Technology (MIT). Over 90% of the students in the course had failed to get a grade of C or better in a previous attempt and were taking the course a second time. Our experimental method was to split a class of approximately 80 students into two equally skilled halves. The two halves were balanced according to data such as gender, planned major, the previous physics and calculus experience, and the scores on myCyberTutor problems attempted during the first six weeks of the course. Consequently, the two halves would experience approximately equal difficulties in a given problem. Furthermore, we established that both halves had the same average skill level taking into account all the myCyberTutor problems (247 in total) given during the semester.

Opposite pedagogical strategies were used in giving each tutorial-related problem pair to the two halves. The “tutorial-first” (TF) group had to finish the tutorial problem before being allowed to attempt the related problem(s). The “problem-first” (PF) group solved the related problem(s) before accessing the tutorial problem. In the case of the torque problem (first problem group), the TF group solved the problems in the order T1 → R1A → R1B while the PF group solved the problems in the order R1B → R1A → T1. For maximum balance the TF and PF groups were alternated between the two pre-selected halves of the class.

If the TF group has significant less difficulty in solving the related problem(s) than the PF group, we can conclude that solving the tutorial problem first helped the TF group to construct a problem-solving schema that helped solve the related problem(s). Conversely, if the PF group has significant less difficulty in answering the questions in the tutorial problem than the TF group, we can conclude that solving the related problem(s) first facilitate the construction of problem-solving schema.

Variables

Students' performance is measured first by the raw "scores" obtained in each of the several variables (see Table 1) which are known to be good predictors of the students' score on traditional assessment instruments such as final exams (Pritchard, Morote, & Kokorowski, 2003), and, second, by the "relative difficulty algorithm," an algorithm we have developed based on getting the maximum reliability using a combination of these scores.

Over 95% of the students eventually reached the solutions to the above three groups of problems using hints, sub-problems and feedback. Therefore, whether or not a student has eventually submitted the correct answer to a given problem is not used as a variable to distinguish the two groups of interest in this study except in considering the skill balance between them. Similarly, the variable, number of "solutions requested" (s) is not considered explicitly since less than 10% of the students requested solutions. However, this variable is taken into account in the difficulty algorithm.

Metacognitive feedback (Mevarech & Kramarski, 1997) variables such as hints indicate areas where students realize that their understanding of concepts or procedures is deficient. Thus, the variable, number of “hints requested” (h) is highly relevant to this study, and represents the students’ confidence in solving a problem. The tutorial-first group should request a significantly lesser number of hints in solving the subsequent problem(s) if the tutorial increases their confidence.

The variable number of responses “correct on first try” (cft) is used since it was found (Pritchard & Morote, 2002) that it is very strongly correlated with standard assessments. Finally, the time variable (t) is defined as the time interval between first calling the problem to first submitting a response, either correct or incorrect. Hence, we will call the time variable (t) as the latency time.

All variables except the variables cft and t were re-scaled from 0 to 1 by using the formula $(\nu_i - \nu_{min})/(\nu_{max} - \nu_{min})$, where ν_i refers to the variable of interest for a particular (i^{th}) student while ν_{max} and ν_{min} refer to the corresponding maximum and the minimum for that variable by any student on a given problem irrespective of the group. The variable cft naturally falls within the range 0 and 1 since it is calculated as the number of parts correct on first try divided by the number of total parts attempted.

Since time intervals on tasks are typically log-normally distributed (Law & Kelton, 1991) they are converted to a logarithmic time, which is often observed to be approximately normally distributed. We thus define a “time score” (t_s) for the i^{th} student on a particular problem as,

$$t_s = \{[\ln (t_i) - \text{Average}(\ln (t))] / 10\sigma \} + 0.5 ,$$

where t is measured in minutes and σ is the standard deviation of ln(t). The average and the standard deviation of the time score are then 0.5 and 0.1, respectively. It should be noted that the variables incorrect answers (i), 1-cft, h, s, and t_s are all measures of difficulty since a larger value of such a variable is indicative of a student having more difficulty with a given problem.

Relative difficulty algorithm

To measure the degree of difficulty that a student had with a problem, we developed a relative "difficulty algorithm" (RDA) with maximum statistical reliability as applied to all the problems done throughout the semester. The reliability of the algorithm was tested using the split-half method, where the problems are divided into two sub-sets of equal content. If the resulting sub-sets each determined difficulty with perfect reliability, each student would receive the same score on both sub-sets and the reliability would be 1.0 (Pritchard & Morote, 2002). The difficulty algorithm was optimized using a computer program that allows successive iterations of the coefficients of variables ia, ina, h, s, t_s and cft to vary until the equation reaches the maximum reliability, which, in this instance was 0.989.

Previous research (Morote & Pritchard, 2002) has shown that the incorrect answers (that is i, which is the sum of ia and ina) correlated most strongly with poor performance on standard assessments. Therefore, we have constrained the sum of the coefficients of incorrect answers (ia and ina) to be equal to 1.0 in the relative difficulty algorithm. Because many extraneous factors can influence the time it takes for students to respond to a problem, we limited the coefficient of t_s-0.35 to be less than or equal 0.75

and found that the algorithm is optimized at 0.75. The other variables were freely varied. The above constraints resulted in the following difficulty algorithm:

Relative “difficulty algorithm” = $0.44(ia) + 0.56(ina) + 0.25(h) + 0.12(s) + 0.75(t_s - 0.35) + 0.20(1 - cft)$.

The relative difficulty reaches its maximum difficulty (2.05) when cft is 0 and all other variables are equal to 1. A student answering with 1.5σ less time score than average and with no mistakes, hints or requested solutions would have zero difficulty. We believe that a negative difficulty could be indicative of cheating.

Findings

The measure of interest is the improvement (Improv.) as measured by the individual difficulty variables i, 1-cft, h, t_s, and the improvement as measured by the relative difficulty algorithm. A positive improvement in the tutorial-first approach (Table 3) implies a decreased average difficulty for the TF group whereas a positive improvement in the problem-first approach (Table 4) implies a decreased average difficulty for the PF group. In the tutorial-first approach, there is improvement in the individual variables in all but one case. The improvement, in individual difficulty variables was significant ($p \leq .05$) in eight and substantial ($.05 < p \leq .1$) in three of the sixteen cases.

On the other hand, in the problem-first approach, the improvement in individual difficulty variables was significant in only two of the twelve cases. Negative improvements were also observed in two cases. The PF group improved ($p \leq .1$) in terms of hints in T1 and the time taken to complete the problems in T2 and T3. In the first problem group the two related problems (R1A and R1B) seems to have helped the PF group understand the tutorial problem (T1) as measured by the difficulty algorithm.

As measured by the relative difficulty algorithm, the first finding is that the relative difficulty of a related problem or a tutorial administered to each group was always reduced by working a preparatory tutorial or a related problem. The second finding is that the reduction in relative difficulty of related problems due to working a tutorial first was greater than the reduction in difficulty on the tutorials due to working a related (self-tutoring) problem first. It should be noted that in Figures 1, 2, and 3, since the variables i, 1-cft, h, t_s, and the relative difficulty (from the relative difficulty algorithm) are measures of difficulty, the improvements on those variables should be interpreted as a decrease in difficulty in the respective variables. The results show that the strategy of using a tutorial-first approach on learning did help students significantly on the development of relevant problem-solving schema over a problem-first approach.

It should be mentioned that using the mean values for the variables of interest, two-tailed *t*-test results showed that the two groups remained well balanced (see Table 2). The difference in students’ performance on the tutorial vs. the related problems was analyzed by applying one-tailed *t*-tests for the variables of interest (the equality of variance and hence the validity of the *t*-test results been established by the Levene test (Wright, 1997)). One-tailed test is used because our primary concern is with the magnitude of the differences rather than with their existence in absolute terms (Garret, 1966).

Conclusion and discussion

Using myCyberTutor, a web-based physics tutor in the introductory calculus-based physics course at MIT we have studied which instructional strategy, tutorial-first or problem-first, helps students in terms of reduction in difficulty in solving subsequent problems. We tested these two instructional approaches (see Table 5) using three kinds of tutorials; a tutorial that helps learn a procedure (T1), a tutorial that helps understand a concept (T2), and a tutorial that helps express a formula (T3) together with the corresponding related problem(s).

The central conclusion of this study is that, in all the cases, the use of tutorial type problems which guide students step by step through the concepts or procedures significantly reduces the difficulty of the subsequent related problem(s) (Table 5). This indicates a superiority of the tutorial-first approach to instruction, at least in the acquisition of basic skills and/or knowledge (Reif & Scott, 1999). On the other hand, the problem-first approach where students completely worked on a typical non-tutorial problem first in a given topic with the help of extensive hints in an interactive online environment did not enable them to internalize the general principles involved. This may result from a previous finding, that novices without guidance frequently become disoriented (Didierjean, 2003; Jonassen, 1988; Tripp & Roby, 1994) and are often unable to allocate their limited resources effectively in order to construct meaning, most likely due to insufficient prior knowledge (Ahn, Brewer, & Mooney, 1992).

Given that the difficulty algorithm is linear in all of the contributing variables except for the time variable (which is logarithmic), the difficulty of the related problem(s) is reduced typically by 25% in working the step by step tutorial first. This corresponds to effect sizes (effect size = improvement / *SD*, where *SD* is the standard deviation of the score in difficulty algorithm of the PF (TF) group for the tutorial-first (problem-first) approach) of 0.57 and 0.22 for the tutorial-first and the problem-first approach, respectively, indicating that the TF group clearly benefit from the preparatory tutorial problem (Table 6, Figures 4, 5).

We interpret the reduction in difficulty to improvement of the students' schema for problem solving. The significant reduction in difficulty in the torque problem under the problem-first approach may well reflect the fact that students are more likely to acquire problem schema under problem-first pedagogy if two prior analog problems were presented instead of just one (Gick & Holyoak, 1983), as well as the sameness of the level of abstraction needed in transferring knowledge from the source to the target problems (Chen, 2002). The significant improvements in "time score" for the tutorial-first approach compared to that of the problem-first approach may result from "saving time" by executing previously learned rules compared to the time needed in learning new rules (Bovair, Kieras, & Polson, 1990; VanLehn, 1996).

We have argued that the students are learning schema. We now argue that they are not just gathering factual knowledge. If the same facts were necessary to answer some part of a tutorial and related problem, the students would learn from the first preparation (when over 90% got it eventually correct) and show equal improvement the second time this factual knowledge was needed. Therefore at least the additional improvement in performance of the TF group on the related problem over the performance of the PF group on the tutorial problem must result from some deeper acquisition of knowledge – that is by building a relevant schema.

It is important to note that we did not check for the persistence of this schema. A future generalization of this work would be to study the decay of the positive effects of the introductory tutorials due to working unrelated problems in between or by delaying the subsequent problems until the next assignment (less than a week in a typical online homework situation). In light of analogical transfer (Gick & Holyoak, 1983) it is also important to determine whether the problem-first approach, although it seems slower, might have advantages for long term retention. In addition we did not consider the possibility that there is some fundamental reason that students find similar questions more difficult to answer (not simply more difficult) if they are in a tutorial environment.

A major confounding factor in studies of this sort is that the time on task is known to be closely related to learning (Brophy, 1988). To the extent that the tutorial problems are longer than the related practice problems, this may give more time for the students to reflect and construct knowledge. We questioned whether it is the time spent on the first (preparatory) problem that is responsible for the decreased difficulty with the subsequent problem(s). To quantify this criterion, we tabulate not the improvement, but the improvement per unit time spent on the preparatory problem (Table 6). We see that the learning per unit time in the tutorial-first approach is twice that of the problem-first approach. Note that an improvement in relative difficulty by some amount represents a fractional rather than an absolute reduction in the number of mistakes. Thus, it is closely analogous to the normalized gain which was shown to depend on the quality of instruction independent of the absolute number or percentage of errors (Hake, 1998).

Since over 90% of the students have taken this course previously but failed to obtain a grade C or better, there may be some lack of generality in the results from the special nature of the class involved. They are definitely below the average MIT level in physics interest. However, their pre and post scores on the Mechanics Baseline Test (MBT- Hestenes & Wells, 1992) are statistically indistinguishable from the Fall class. Furthermore, they measure slightly above average for introductory classes in universities (Morote & Pritchard, 2002) as measured by Force Concept Inventory (Hestenes, Wells, & Swackhamer, 1992) and the MBT. Overall, we feel that they are generally representative of good university students in a calculus-based introductory course, except that they lack the intrinsic interest in the subject to think beyond getting the answer.

This study shows that the impact of computer tutorials on student learning depends on the pedagogical strategy that is followed. The tutorial-first approach was shown to be superior to that of the problem-first approach for the development of problem-solving schema. The tutorial-first approach was shown to make subsequent problems easier to solve and gave students increased confidence and comprehension in solving the related problem(s) as measured by relative difficulty, ia, ina, h, s, cft, and t_s.

Finally, we emphasize the unobtrusiveness of our investigative technique and its statistical robustness. The technique is an example of integrated assessment – the assessment is based entirely on observing the student in the learning environment without the necessity of tests or assessment protocols that may distract from the learning experience. It closely resembles the ideal form of assessment – direct observation (Gronlund, 2001). Assessment within the tutorial environment is sufficiently insightful that the individual assessment variables such as number of hints, incorrect answers, time for response, and correct the first time generally show statistical significance by themselves. The relative difficulty algorithm is a good summary of the individual variables but its use is not necessary to obtain statistically significant results.

Our overarching conclusion is that students significantly improve their problem-solving schema when they are given an online tutorial problem in which the lesson content is sequenced and interactive. In this regard our study supports the observed increased ability in giving correct solutions using computer-based tutorials in a reciprocal instructional strategy (Reif & Scott, 1999). Since the web-based tutor used in this study has the overall instructional pedagogy of providing feedback, and since such feedback is an essential instructional component in facilitating schema development (Price & Driscoll, 1997), we argue that both the tutorial-first and the problem-first pedagogies are superior to that of standard homework which has none. Such an advantage over standard homework may result from providing opportunities for immediate tuning (where students become aware of inadequacies of their existing schema) and restructuring (where new schemas that addresses the inadequacies are integrated with the old schemas) to students via feedback. Conversely, our finding that the problem-first pedagogy is less effective than tutorial-first pedagogy, combined with the expectation that standard homework (without hints and immediate feedback) should be even less effective at facilitating schema development, confirms the widespread feeling in the physics education community that standard homework does not impart problem-solving schema effectively (K. Heller, personal communication, August 6, 2003).

We have provided evidence of the tutorial-first approach as an effective approach to learn problem-solving in physics in a web-based environment. Hence, the use of hints and sub-problems in this particular setting of e-learning (Clark & Mayer, 2003) is most effective in a tutorial-first pedagogy. Until further studies contradict or qualify our findings (e.g. by showing that the positive effect does not persist), we recommend using online tutorials in the old fashioned way – that is, to employ tutorial-first instructional strategy to present new problem-solving schema to students.

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Appendix A

Problems: Learning a procedure

T1: Torque about the z Axis

Learning Goal: To understand two different techniques for computing the torque on an object due to an applied force.

Imagine an object with a pivot point p at the origin of the coordinate system shown. [see Figure A1.]

The force vector F lies in the xy plane, and this force of magnitude **F** acts on the object at a point in the xy plane. The vector **r** is the position vector relative to the pivot point p to the point where **F** is applied.

The torque on the object due to the force **F** is equal to the cross product $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$. When, as in this problem, the force vector and lever arm both lie in the xy plane of the paper or computer screen, only the z component of torque is nonzero.

When the torque vector is parallel to the z axis ($\boldsymbol{\tau} = \tau\, \mathbf{z}$), it is easiest to find the magnitude and sign of the torque, τ, in terms of the angle θ between the position and force vectors using one of two simple methods: the *Tangential Component of the Force* method or the *Moment Arm of the Force* method.

Note that in this problem, the positive z direction is perpendicular to the computer screen and points toward you (given by the right hand rule $\mathbf{x} \times \mathbf{y} = \mathbf{z}$), so a positive torque would cause counterclockwise rotation about the z axis.

Tangential component of the force

Part A [see Figure A2.]

Decompose the force vector **F** into radial (i.e., parallel to **r**) and tangential (perpendicular to **r**) components as shown. Find the magnitude of the radial and tangential components, F_r and F_t. You may assume that θ is between zero and 90 degrees. Enter your answer as an ordered pair. Express F_t and F_r in terms of F and θ.

Part B

Is the following statement true or false?

The torque about point p is proportional to the length r of the position vector **r**.

O true O false

Part C

Is the following statement true or false?

Both the radial and tangential components of **F** generate torque about point p.

O true O false

Part D

Is the following statement true or false?

In this problem, the tangential force vector would tend to turn an object clockwise around pivot point p.

O true O false

Part E

Find the torque τ about the pivot point p due to force **F**. Your answer should correctly express both the magnitude and sign of τ. Express your answer in terms F_t and r or in terms of F, θ, and r.

Moment arm of the force

In the figure [see Figure A3], the dashed line extending from the force vector is called the line of action of **F**. The perpendicular distance r_m from the pivot point p to the line of action is called the moment arm of the force.

Part F

What is the length, r_m, of the moment arm of the force **F** about point p? Express your answer in terms of τ and θ.

Part G

Find the torque τ about p due to **F**. Your answer should correctly express both the magnitude and sign of τ. Express your answer in terms of r_m and F or in terms of r, θ, and F.

R1A: Torque Practice – 1

In this problem you practice finding torques using the two most common methods for finding the torque in two-dimensional problems (where the force vectors and origin all lie in the same plane).

The figure [see Figure A4] shows a 10 Å – 10 grid. The squares have unit length, so you can find or calculate the distance to point of application of the force or the moment arm. Don’t use this scale for the magnitude of the forces – their magnitudes are given by their respective symbols. Thus a torque, which is equivalent to a distance times a force, would look like 5*Fa, or perhaps 3*Fb*sin(alpha). Also, in entering answers, the angle between the vector labeled Fe and the x-axis must be entered as “theta” and the angle between the vector labeled Fc and the x-axis must be entered as “phi”.

Torques must be found around a given point or origin, in this case the point shown as solid dot on the figure. Don’t forget that counterclockwise torques are positive as the circular arrow in the lower left of the figure reminds you.

You are to find the torques by one of two methods.

Part A

Find the torque about the origin due to force Fa.

Part B

Find the torque about the origin due to force Fb.

Part C

Find the torque about the origin due to force Fc. Remember to use “phi” for the angle.

Part D

Find the torque about the origin due to force Fd.

Part E

Find the torque about the origin due to force Fe. Remember to use “theta” for the angle.

Part F

Find the torque about the origin due to force Ff.

R1B: Torque Practice – 2

In this problem you practice finding torques using the two most common methods for finding the torque in two-dimensional problems (where the force vectors and origin all lie in the same plane).

The figure [see Figure A5] shows a 10 Å – 10 grid. The squares have unit length, and the forces have magnitudes given by their respective symbols (NOT by the length of the drawn vector). The vectors are drawn to scale in direction, but not in magnitude. Thus a torque, which is equivalent to a distance times a force, would look like 5*Fa, or perhaps 3*Fb*sin(alpha).

Torques must be found around a given point or origin, in this case the point shown as a solid dot on the figure. Don't forget that counterclockwise torques are positive as the circular arrow in the lower left of the figure reminds you.

You are to find the torques by one of two methods, which we will label 1 or 2:

1. Using the moment arm.
2. Using the Tangential (Perpendicular) force.

Part A

Find the torque about the origin due to force Fa.

Part B

Find the torque about the origin due to force Fb.

Part C

Find the torque about the origin due to force Fc; give your answer in terms of Fc and theta.

Part D

Find the torque about the origin due to force Fd; give your answer in terms of Fd and phi.

Part E

Find the torque about the origin due to force Fe.

Appendix B

Problems: Understanding a concept

T2: Newton's 3rd Law Discussed

Learning Goal: To understand Newton's 3rd law, which states that a physical interaction always generates a *pair* of forces on the two interacting bodies.

In *Principia*, Newton wrote:

To every action there is always opposed
an equal reaction: or, the mutual actions
of two bodies upon each other are always
equal, and directed to contrary parts.

(translation by Cajori)

The phrase after the colon (often omitted from textbooks) makes it clear that this is a statement about the nature of force. The central idea is that physical interactions (e.g., due to gravity, bodies touching, or electric forces) cause forces to arise between *pairs* of bodies. Each pairwise interaction produces a *pair* of opposite forces, one acting on each body. In summary, each physical interaction between two bodies generates a *pair* of forces. Whatever the physical cause of the interaction, the force on body A from body B is equal in magnitude and opposite in direction to the force on body B from body A.

Incidentally, Newton states that the word "action" denotes both (a) the force due to an interaction and (b) the changes in momentum that it imparts to the two interacting bodies. If you haven't learned about momentum, don't worry, for now this is just a statement about the origin of forces.

Mark each of the following statements as true or false. If a statement refers to “two bodies” interacting via some force, you are *not* to assume that these two bodies have the same mass.

Part A

Every force has one and only one 3rd law pair force.

O true O false

Part B

The two forces in each pair act in opposite directions.

O true O false

Part C

The two forces in each pair can act on the same body or on different bodies

O true O false

Part D

The two forces in each pair may have different physical origins (for instance, one of the forces could due to gravity, and its pair force could be a normal contact force).

O true O false

Part E

The two forces of a 3rd law pair *always* act on different bodies.

O true O false

Part F

Given that two bodies interact via some force, the accelerations of these two bodies have the same magnitude but opposite direction. (Assume no other forces act on either body.)

O true O false

Part G

According to Newton's 3rd law, the force on the (smaller) moon due to the (larger) earth is

O greater in magnitude and antiparallel to the force on the earth due to the moon.

O greater in magnitude and parallel to the force on the earth due to the moon.

O equal in magnitude but antiparallel to the force on the earth due to the moon.

O equal in magnitude and parallel to the force on the earth due to the moon.

O smaller in magnitude and antiparallel to the force on the earth due to the moon.

O smaller in magnitude and parallel to the force on the earth due to the moon.

R2: A Book on a Table

A book weighing 5N rests on top of a table [see Figure B1].

Part A

A downward force of magnitude 5N is exerted on the book by the force of

O the table O gravity O inertia

Part B

An upward force of magnitude _________ is exerted on the _________ by the table.

Part C

Do the downward force in Part A and the upward force in Part B constitute a 3rd law pair?

O yes O no

Part D

The reaction to the force in Part A is a force of magnitude _______, exerted on the ________ by the ________. Its direction is _________.

Part E

The reaction to the force in Part B is a force of magnitude ________, exerted on the ________ by the _________. Its direction is _________.

Part F

Which of Newton’s laws dictates that the forces in Parts A and B are equal and opposite?

O Newton’s 1st and 2nd laws

O Newton’s 3rd law

Part G

Which of Newton’s laws dictates that the forces in Parts B and E are equal and opposite?

O Newton’s 1st and 2nd laws

O Newton’s 3rd law

Appendix C

Problems: Expressing a formula

T3: Harmonic Oscillator Kinematics

Learning Goal: To understand the application of the general harmonic equation to the kinematics of a spring oscillator.

One end of a spring with spring constant k is attached to the wall [see Figure C1]. The other end is attached to a block of mass m. The block rests on a frictionless horizontal surface. The equilibrium position of the left side of the block is defined to be $x = 0$. The length of the relaxed spring is L.

The block is slowly pulled from its equilibrium position to some position $x_{init} > 0$ along the x axis. At time $t = 0$, the block is released with zero initial velocity.

The goal is to determine the position of the block $x(t)$ as a function of time in terms of ω and $x_{init.}$

It is known that a *general* solution for the coordinate of a harmonic oscillator is

$x(t) = C \cos(\omega t) + S \sin(\omega t)$,

where C, S, and ω are constants [see Figure C2].

Your task, therefore, is to determine the values of C and S in terms of ω and $x_{init.}$

Part A

Using the general equation for $x(t)$ given in the problem introduction, express the initial position of the block x_{init} in terms of C, S, and ω.

Part B

Find the value of S using the given condition that the initial velocity of the block is zero: $v(0) = 0$.

Part C

What is the equation $x(t)$ for the block?

Now, imagine that we have exactly the same physical situation but that the x axis is translated, so that the position of the wall is now defined to be $x = 0$ [see Figure C3]. The initial position of the block is the same as before, but in the new coordinate system, the block's starting position is given by $x_{new}(t = 0) = L + x_{init}$.

Part D

Find the equation for the block's position $x_{new}(t)$ in the new coordinate system. Express your answer in terms of L, x_{init}, ω, and t.

R3: Position, Velocity, Acceleration of Oscillator

Learning Goal: To learn to find kinematic variables from a graph of position vs. time.

The graph [see Figure C4] of the position of an oscillating object as a function of time is shown.

Some of the questions ask you to determine ranges on the graph over which a statement is true. When answering these questions, choose the *most complete* answer. For example, if the answer "B to D" were correct, then "B to C" would technically also be correct – but you will only receive credit for choosing the most complete answer.

Part A

Where on the graph is $x > 0$?

O A to B

O A to C

O C to D

O C to E

O B to D

O A to B and D to E

Part B

Where on the graph is $x < 0$?

O A to B

O A to C

O C to D

O C to E

O B to D

O A to B and D to E

Part C

Where on the graph is $x = 0$?

O A only

O C only

O E only

O A and C

O A and C and E

O B and D

Part D

Where on the graph is the velocity $v > 0$?

O A to B

O A to C

O C to D

O C to E

O B to D

O A to B and D to E

Part E

Where on the graph is the velocity $v < 0$?

O A to B

O A to C

O C to D

O C to E

O B to D

O A to B and D to E

Part F

Where on the graph is the velocity $v = 0$?

O A only

O B only

O C only

O D only

O E only

O A and C

O A and C and E

O B and D

Part G

Where on the graph is the acceleration $a > 0$?

O A to B

O A to C

O C to D

O C to E

O B to D

O A to B and D to E

Part H

Where on the graph is the acceleration $a < 0$?

O A to B

O A to C

O C to D

O C to E

O B to D

O A to B and D to E

Part I

Where on the graph is the acceleration $a = 0$?

O A only

O B only

O C only

O D only

O E only

O A and C

O A and C and E

O B and D

Tables

Table 1

Variables

Traditional variables	
c	correct answers
i	incorrect answers (= ina + ia)
ina	incorrect answers without receiving advice
Feedback variables	
h	hints requested
s	solutions requested
ia	incorrect answers receiving advice
Immediacy variable	
cft	probability of getting a correct answer on first try
Latency time variable	
t	time to submit the first response (either correct or incorrect)

Note. Traditional and feedback variables are re-scaled to full within the range 0 and 1.

Table 2

Skill balance of TF and PF for the whole semester, and probability that such differences would occur by chance if the groups were in fact equal.

Variable	Difference in Means	*p*
i	.0621	.19
c	-.0063	.89
cft	.0204	.39
h	.0906	.34
s	.0076	.55
t_s	.0178	.47

Table 3

Tutorial-first approach: Effect of tutorial problems on its related problem(s)

T1: Tutorial: Torque z - component					
R1A: Related problem: Torque practice 1	R1A[a]	T1[b] → R1A[c]			
Variable	*M* (PF)	*M* (TF)	Improv.	*p*	
i	.29	.20	.09	.030	**
1-cft	.50	.44	.06	.150	
h	.48	.35	.13	.050	**
t_s	.52	.48	.04	.045	**
Difficulty algorithm	.66	.44	.22	.004	**

(T1+R1A): (Tutorial + Torque practice 1)					
R1B: Related problem: Torque practice 2	R1B[d]	(T1+R1A) → R1B[e]			
Variable	*M* (PF)	*M* (TF)	Improv.	*p*	
i	.29	.20	.09	.028	**
1-cft	.49	.46	.03	.300	
h	.50	.38	.12	.055	*
t_s	.53	.47	.06	.002	**
Difficulty algorithm	.60	.42	.18	.007	**

T2: Newton’s 3rd law					
R2: Book on table	R2[f]	T2[g] → R2[h]			
Variable	*M* (PF)	*M* (TF)	Improv.	*p*	
i	.46	.34	.12	.020	**

1-cft	.39	.31	.08	.040	**
h	.19	.06	.13	.055	*
t_s	.52	.48	.04	.080	*
Difficulty algorithm	.46	.36	.10	.045	**
T3: Harmonic oscillator tutorial					
R3: Position velocity, acceleration of oscillator	R3[i]	T3[j] → R3[k]			
Variable	*M* (PF)	*M* (TF)	Improv.	*p*	
i	.28	.17	.11	.040	**
1-cft	.03	.02	.01	.210	
h	.17	.13	.04	.271	
t_s	.50	.50	.00	.475	
Difficulty algorithm	.33	.26	.07	.065	*

Note. Improv. = Improvement.

[a]n = 27. [b]n = 34. [c]n = 32. [d]n = 27. [e]n = 32. [f]n = 34. [g]n = 32. [h]n = 31. [i]n = 27. [j]n = 26. [k]n = 26.

* $.05 < p \leq .10$. ** $p \leq .05$.

Table 4

Problem-first approach: Effect of related problems on its tutorial problem

T1: Torque z - component

R1B+R1A: Torque practice 1 and 2	T1^{a}	(R1B+R1A)$^{b} \rightarrow$ T1^{c}		
Variable	*M* (TF)	*M* (PF)	Improv.	*p*
i	.26	.24	.02	.38
1-cft	.41	.37	.04	.20
h	.22	.10	.12	.04 **
t_s	.51	.49	.02	.12
Difficulty algorithm	.50	.41	.09	.08 *

T2: Newton's 3rd law

R2: Book on table	T2^{d}	R2$^{e} \rightarrow$ T2^{f}		
Variable	*M* (TF)	*M* (PF)	Improv.	*p*
i	.40	.40	.00	.50
1-cft	.36	.34	.02	.36
h	.00	.00	.00	N/A
t_s	.53	.47	.06	.01 **
Difficulty algorithm	.39	.34	.05	.13

T3**:** Harmonic oscillator tutorial

R3: Position, velocity, acceleration of oscillator	T3^{g}	R3$^{h} \rightarrow$ T3^{i}		
Variable	*M* (TF)	*M* (PF)	Improv.	*p*
i	.24	.22	.02	.37

1-cft	.35	.38	-.03	.35	
h	.37	.39	-.02	.42	
t_s	.52	.48	.04	.09	*
Difficulty algorithm	.51	.50	.01	.45	

Note. Improv. = Improvement.

[a]n = 34. [b]n = 27. [c]n = 34. [d]n = 32. [e]n = 34. [f]n = 34. [g]n = 26. [h]n = 27. [i]n = 29.

***** $.05 < p \leq .10$. ** $p \leq .05$.

Table 5

Comparison of tutorial-first versus problem-first strategy for reduction in difficulty

Tutorial Problem	Related problem(s)	Variable	Tutorial-first T → R	Problem-first R → T
T1: Learning a procedure				
(Torque)	R1A & R1B	i	**	
		cft		
		h	**	**
		t_s	**	
		RDA	**	*
T2: Understanding a concept			**	
(Newton's 3rd law)	R2	i		
		cft	**	
		h	*	
		t_s	*	**
		RDA	**	
T3: Expressing a formula				
(Harmonic oscillator)	R3	i	**	
		cft		
		h		
		t_s		*
		RDA	*	

* $.05 < p \leq .10$. ** $p \leq .05$.

Table 6

Improvement per unit time and effect sizes based on the relative difficulty algorithm

Problem sequence	Preparatory Problem	*Mdn* time (minutes) for the preparatory problem	Learning approach	Improv./*Mdn* time		Effect Size (=Improv. / *SD*)	
				Tut.	Prob.	Tut.	Prob.
T1 → R1A	T1	22.88	Tut.	0.010		0.80	
(R1A+R1B) → T1	(R1A+R1B)	32.50	Prob.		0.003		0.37
T2 → R2	T2	5.34	Tut.	0.019		0.45	
R2 → T2	R2	4.66	Prob.		0.011		0.25
T3 → R3	T3	19.75	Tut.	0.004		0.46	
R3 → T3	R3	3.39	Prob.		0.003		0.03
			Average	0.013	0.006	0.57	0.22

Note. Improv. = Improvement from the difficulty algorithm; Tut. = Tutorial-first; Prob. = Problem-first. *SD* = *SD* of the difficulty algorithm score for PF in tutorial-first. *SD* = *SD* of the difficulty algorithm score for TF in problem-first.

Figure Captions

Figure 1. Improvements in the torque problems for the tutorial-first and problem-first approaches.

Figure 2. Improvements in the Newton’s 3rd law problems for the tutorial-first and problem-first approaches.

Figure 3. Improvements in the simple harmonic motion problems for the tutorial-first and problem-first approaches.

Figure 4. Improvement per unit time spent on preparatory problem.

Figure 5. Effect size on overall difficulty.

Figure A1. Diagram 1 for “Torque – z component” (tutorial problem T1).

Figure A2. Diagram 2 for “Torque – z component” (tutorial problem T1).

Figure A3. Diagram 3 for “Torque – z component” (tutorial problem T1).

Figure A4. Diagram for “Torque practice – 1” (related problem R1A).

Figure A5. Diagram for “Torque practice – 2” (related problem R1B).

Figure B1. Diagram for “A book on a table” (related problem R2).

Figure C1. Diagram 1 for “Harmonic oscillator kinematics” (tutorial problem T3).

Figure C2. Diagram 2 for “Harmonic oscillator kinematics” (tutorial problem T3).

Figure C3. Diagram 3 for “Harmonic oscillator kinematics” (tutorial problem T3).

Figure C4. Diagram for “Graphical position, velocity, acceleration of oscillator” (related problem R3).

Figures

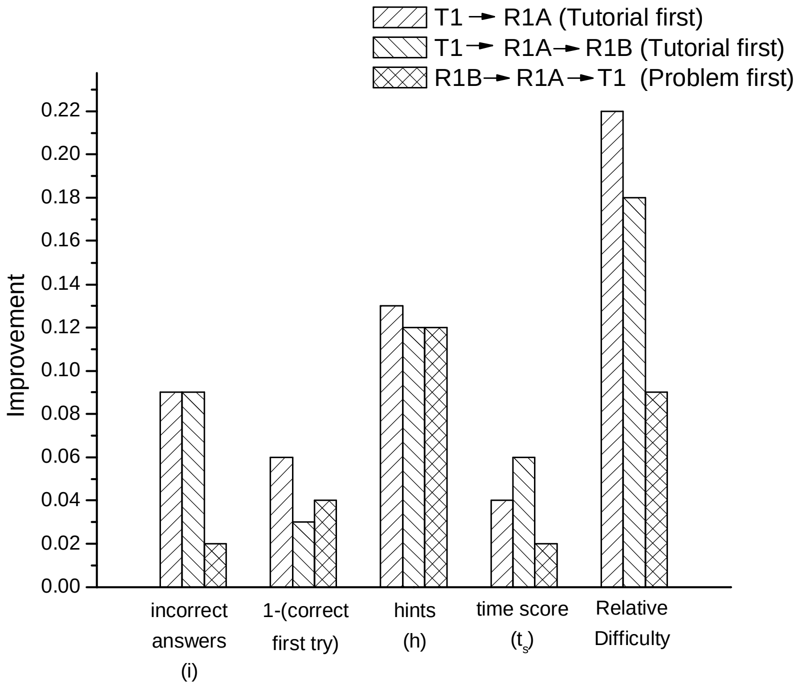

Figure 1. Improvements in the torque problems for the tutorial-first and problem-first approaches.

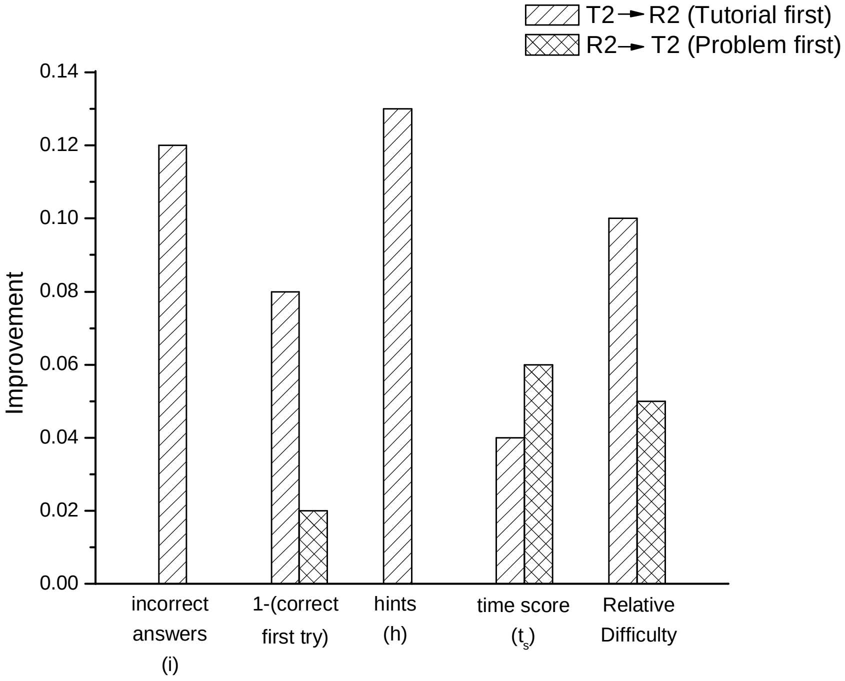

Figure 2. Improvements in the Newton’s 3rd law problems for the tutorial-first and problem-first approaches.

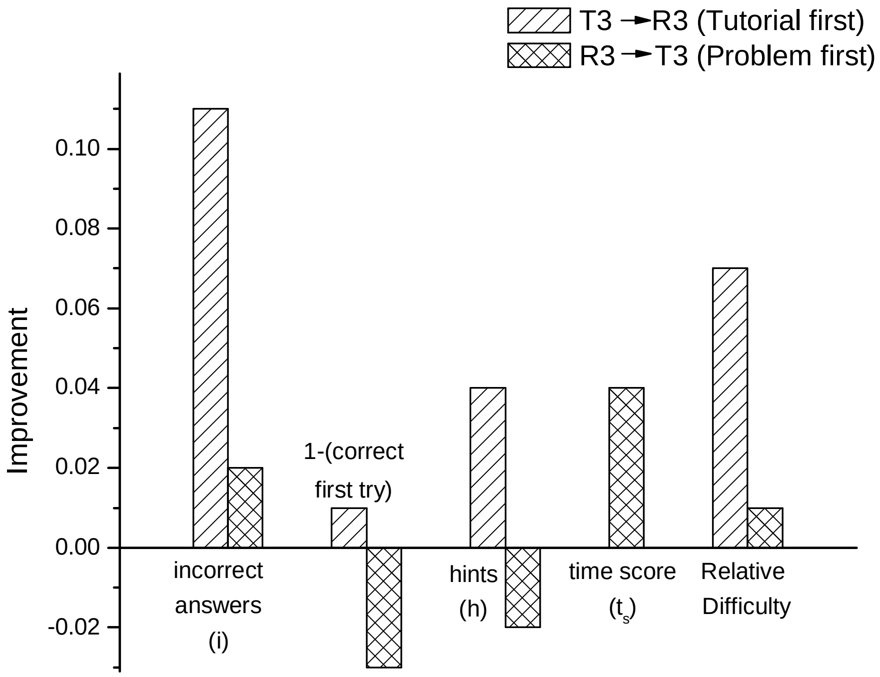

Figure 3. Improvements in the simple harmonic motion problems for the tutorial-first and problem-first approaches.

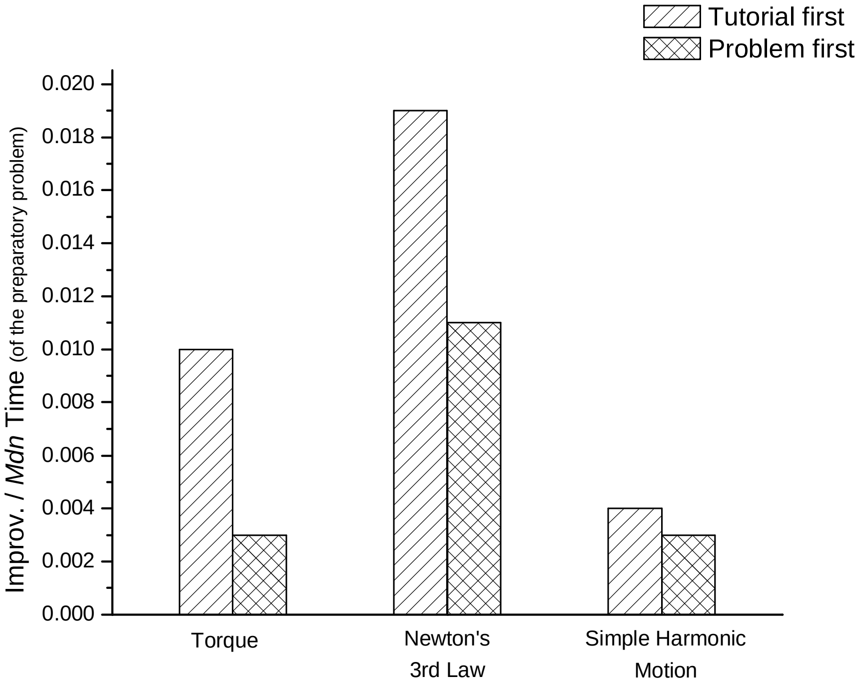

Figure 4. Improvement per unit time spent on preparatory problem

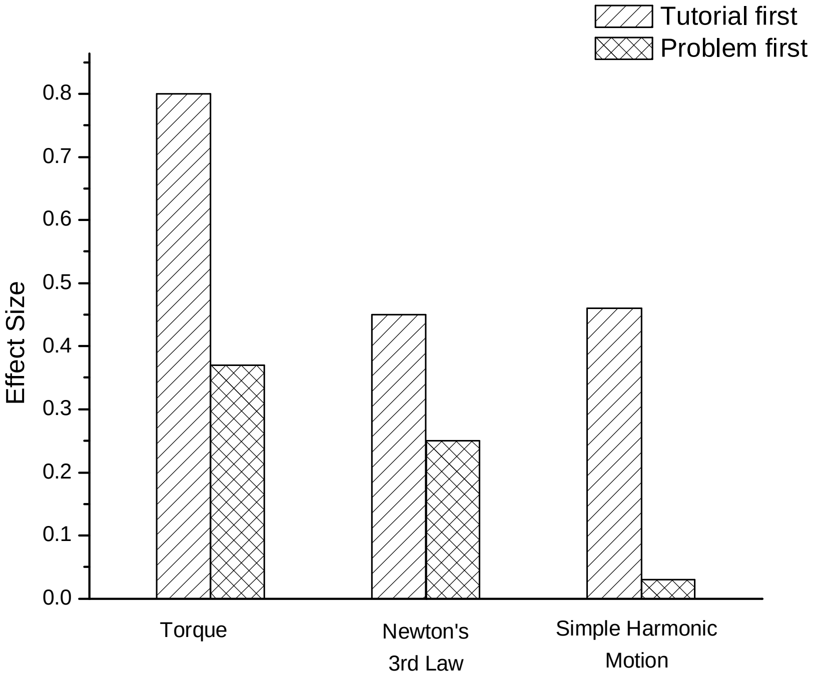

Figure 5. Effect size on overall difficulty

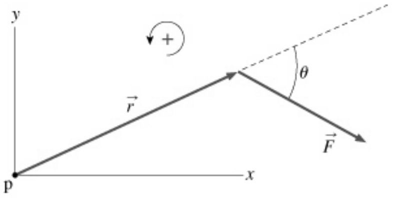

Figure A1. Diagram 1 for “Torque – z component” (tutorial problem T1).

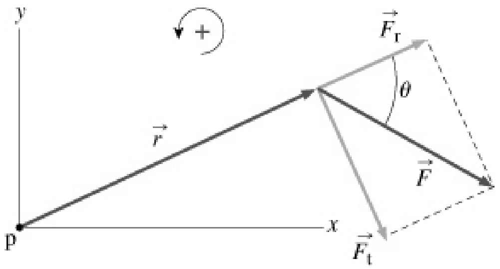

Figure A2. Diagram 2 for “Torque – z component” (tutorial problem T1).

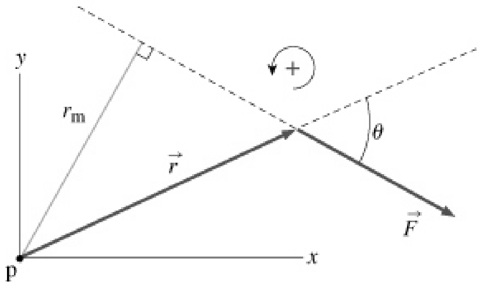

Figure A3. Diagram 3 for “Torque – z component” (tutorial problem T1).

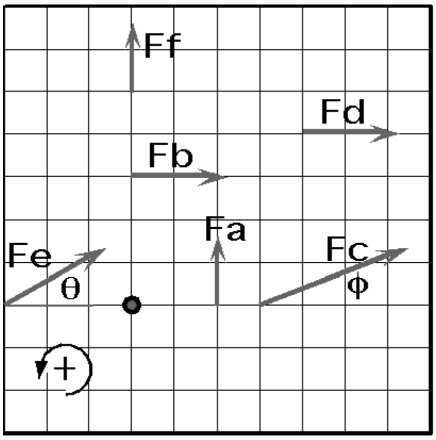

Figure A4. Diagram for “Torque practice – 1” (related problem R1A).

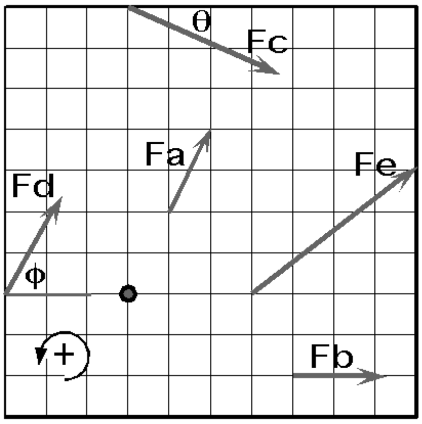

Figure A5. Diagram for “Torque practice – 2” (related problem R1B).

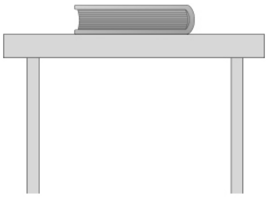

Figure B1. Diagram for “A book on a table” (related problem R2).

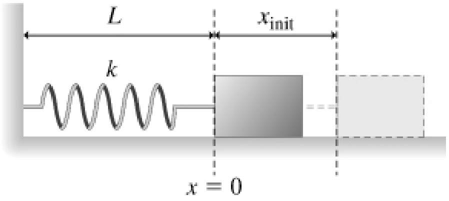

Figure C1. Diagram 1 for “Harmonic oscillator kinematics” (tutorial problem T3).

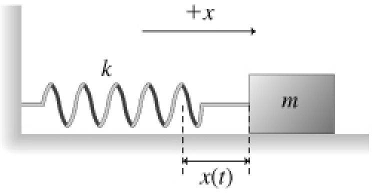

Figure C2. Diagram 2 for “Harmonic oscillator kinematics” (tutorial problem T3).

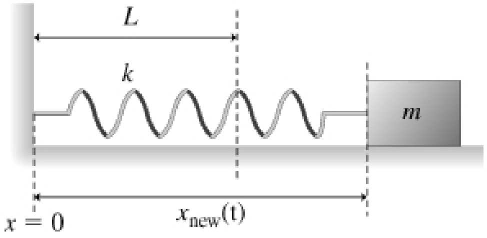

Figure C3. Diagram 3 for “Harmonic oscillator kinematics” (tutorial problem T3).

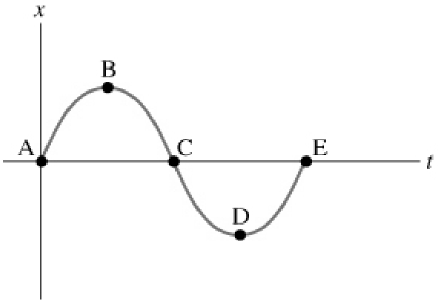

Figure C4. Diagram for “Graphical position, velocity, acceleration of oscillator” (related problem R3).